ELLIOTT 903

Volume 2: PROGRAMMING INFORMATION

Part 2: PROGRAM DESCRIPTIONS

Section 16: QDAATAN (B. 105A)

Contents

Chapter 1: INTRODUCTION

1.1 Purpose

To calculate, as a double-length fraction

$$t = (1/\pi) \tan^{-1} (x/y)$$

and $b = (1/2\pi)$ true bearing,

where x, y are double-length fractions.

1.2 Form of Distribution

The program is distributed as a SIR mnemonic tape.

1.3 Method of Use

The routine is assembled as a block of the user's program and entered as a sub-routine. It can be run at any program level and in any store-module.

When QDAATAN is used QDLA must also be held in store.

1.4 Accuracy

The maximum error is 2^{-34} (0.6×10^{-10})

Chapter 2: FUNCTIONS

2.1 Notation

x(m.s.) = most significant half of x
x(l.s.) = least significant half of x

x, t are as defined in 1.1

2.2 Format

A double-length fraction, x, is held in two consecutive store locations, X and X+1.

Bit 18 of X gives the sign of x
Bits 17-1 of X give the 17 most significant bits of x
Bit 18 of X+1 must be 0
Bits 17-1 of X+1 give the 17 least significant bits of x.

Negative number representation is by the usual 2's complement notation (except that bit 18 of X+1 must be 0).

2.3 Number Type

All numbers must be treated by the programmer as pure fractions.

To enable this to be done QDAATAN calculates

$t = (1/\pi)\ \tan^{-1}(x/y)$

Note, therefore, that t is the value of an angle as a fraction of π radians (180°).

2.4 Entry and Exit

A double-length number occupies two consecutive locations; only the first is given below.

Entry (for assembly by SIR)

Place x in QDAATAN+136
y in QDAATAN+138
and enter 11QDAATAN
8QDAATAN+1

Exit

t in QDAATAN+142
b in QDAATAN+146
b(m.s.) in the accumulator

Note. The true bearing is found by taking
x along the easterly axis
y along the northerly axis
and measuring the angle in a clockwise direction.
N.B. The instruction pair must not form part of a pseudo-program interpreted by QDLA.

2.5 Identifiers

QDAATAN must be declared as a global identifier in all blocks of a SIR program which refer to it.

Chapter 3: ERROR INDICATION

If x=y=0
then 00000.001 is output continuously.

Chapter 4: METHOD USED

QDAATAN uses QDLA to interpret some of the double-length calculations.

a) The program computes

$$a = \begin{cases} |x/y| & \text{if } |x/y| < 1 \\ |y/x| & \text{if } |x/y| \geq 1 \end{cases}$$

and applies the transformation

$$z = \frac{a - (\sqrt{2} - 1)}{(3-2\sqrt{2})\,a + (\sqrt{2} - 1)}$$

Note $|z| < 1$

b) $s = 1/\pi \tan (\sqrt{2}-1)z$

is calculated by a Chebyshev series.

c) The final result is found by forming

$$u = (1/\pi) \tan^{-1} |x/y| = \begin{cases} \frac{1}{8} + s & \text{for } |x| < |y| \\ \frac{1}{2} - (\frac{1}{8} + s) & \text{for } |x| \geq |y| \end{cases}$$

and t is found according to the table below

	$y \geq 0$	$y < 0$
$x/y \geq 0$	u	u-1
$x/y \leq 0$	1-u	-u

and $b = \begin{cases} \frac{1}{2}t & \text{if } x \geq 0 \\ \frac{1}{2}t+1 & \text{if } x < 0 \end{cases}$

Chapter 5: TIME TAKEN

Approximately 42.4 milliseconds.

Chapter 6: STORE USED

QDAATAN uses 167 consecutive locations and the appropriate B-register.